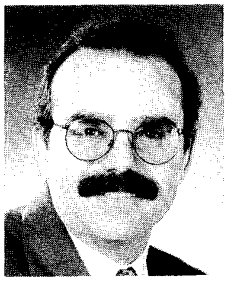

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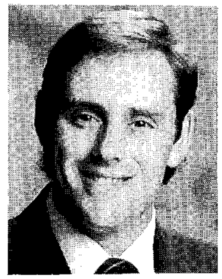

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Foreword

The generation of monostatic RCS data is a computationally intensive task. This month's column addresses some practical aspects of a method by which the monostatic RCS can be approximated from bistatic RCS data, which is less expensive to compute. At the core of this is the Monostatic-to-Bistatic Equivalence Theorem. This is an approximate result, only valid up to a certain angle: Maj. Wilson presents a novel methodology for predicting this angle. His approach is based on a metric, which takes into account target complexity.

The backlog of papers for this column is presently quite small. If you have an interesting idea, or practical observations about developing CEM codes, or experience with a new programming methodology/environment, which EM programmers would find of use, we encourage you to submit it for our column.

Method for Predicting the Maximum Reliable Angle to Use in the Monostatic -to-Bistatic Equivalence Theorem

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Abstract

A metric is proposed that allows reliable prediction of the maximum bistatic angle for which the Monostatic-to-Bistatic Equivalence Theorem (MBET) can be used. Currently, the theorem leaves the term "sufficiently smooth" undefined, making selection of the maximum angle somewhat subjective. The proposed metric provides a quantitative evaluation of complexity/smoothness, and relates this to an angle limit based on an empirically derived statistical error profile. That is, the metric allows prediction of the maximum bistatic angle for which the MBET provides less than a 1.5 dB error at a confidence level of 95%. Although the metric is presently only demonstrated for two-dimensional (2D) objects at a single polarization and error value, sufficient experiments following the same process can easily extend the method to three-dimensional objects, arbitrary polarizations, and alternate error tolerances. Such a capability allows for optimization of either monostatic collections used for prediction of bistatic data sets, or bistatic computations interpolated to monostatic results.

Keywords: Multistatic scattering; radar cross sections; radar data processing; radar measurements; radar scattering; complexity theory; monostatic to bistatic equivalence theorem; computational electromagnetics

1. Introduction

The Monostatic-to-Bistatic Equivalence Theorem (MBET) is a useful tool for reducing the computation time for monostatic radar cross section (RCS) predictions by allowing interpolation of more easily computed bistatic data. This is important to the field of computational electromagnetics (CEM), since it has the potential to reduce run times for large objects.

The simplest version of the MBET states, "...the bistatic RCS of a sufficiently smooth, perfectly conducting target is equal to the monostatic RCS measured on the bisector of the bistatic angle" [1]. Although worded for estimation of bistatic scattering from an available monostatic data set, the equivalence goes both ways. A graphical depiction of the interpolation procedure is shown in Figure 1.

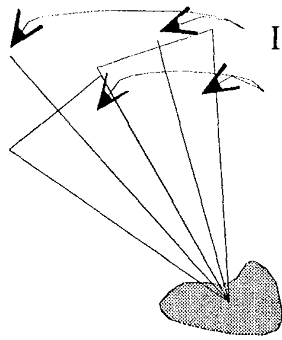

Figure 1. The use of Monostatic-Bistatic Equivalence Theorem (MBET) in computational electromagnetics (CEM).

For quasi-exact CEM techniques, the determination of surface currents is typically one of the more lengthy process steps. Once these currents have been found for a single illumination angle, the calculation of bistatic scattering is fairly rapid. The ability to reduce the number of separate illumination angles while using the MBET to preserve the desired number of observation angles can provide significant savings in run times. This approach, however, introduces a need to balance speed and accuracy. Liberal use of the MBET over wide bistatic will result in fewer illumination angles, but errors can increase rapidly for complex objects [2]. Very limited use of the MBET or no use at all can prevent such interpolation errors. Unfortunately, there is a real possibility that a significant portion of the run time will be unnecessary for achieving a practical accuracy requirement.

A method for predicting MBET performance is then useful for guiding determination of the interpolation-interval size. This effort proposes a method for predicting the maximum reliable interpolation angle when given a specific error tolerance. RCS is essentially random for most targets that are large or complex enough to warrant study, and concave shapes pose special challenges to the MBET. Errors must then be treated statistically: the tolerance cannot be absolute but will rather have a level of confidence. The prediction process is demonstrated for two-dimensional (2D) objects with an error limit of 1.5 dB at a 95% confidence level. That is, use of the suggested bistatic limit will result in 95% of the RCS values interpolated at this maximum angle solving to within 1.5 dB of the true monostatic value. It is possible to have jumps between different MBET interpolation intervals of 3 dB while still meeting the 1.5 dB limit, since the error is given with respect to the true monostatic value, rather than discontinuities in the interpolated RCS pattern. The metric also allows for different bistatic-angle recommendations at different observation angles for oblong objects.

Results shown were obtained using an electric-field integral equation (EFIE) Method of Moments (MoM) calculation. The process is easily repeatable for any arbitrary polarization, error tolerance, or level of confidence, so long as an appropriate number of experiments are run in order to populate the required database. The error profile is then drawn empirically from the database, which must contain sufficient MBET error data for projecting significant values.

2. Approach

A total of 166 objects, with perfect-electric-conductor (PEC) surfaces, extending up to 25 wavelengths (λ) in diameter (the majority of which contained sharp edges and concavities), were placed through the following process:

1. The monostatic RCS was found around each object at 0.1° intervals, using the two-dimensional EFIE MoM. This data formed the *truth model* for each object.

2. The bistatic RCS was found at 0.1° intervals for each illumination angle at 1° increments, around the object, resulting in a series of *test sets*.

3. For each of the *test sets*, MBET results were compared with the monostatic *truth-model* values for incrementing bistatic angles until the error limit was first reached. This was done for both directions: clockwise and counter-clockwise angle sweeps.

4. For each of the *test sets*, the last clockwise and counter-clockwise angles were recorded just before the error limit was breached. This gave two angle recommendations with 0.1° resolution for each illumination angle. When the data for all targets were compiled, no meaningful differences existed between the different directions.

5. For each illumination angle, for each object, a value of the complexity metric was found, as described in the following section. The metric used as the reference value was the product of the computed complexity value and the maximum cross-range dimension in wavelengths of the target at that particular aspect angle.

6. After all data had been collected, the maximum angles were grouped according to the complexity metric value. The total data set included 119,520 points.

7. The metric values were grouped into several bins. In each of these, the fifth-percentile angle was found and reported as the recommended maximum for that specific complexity-value range. This meant that for a specific metric value, 95% of the data points in the appropriate bin indicated that a larger angle could be used than was reported, while achieving less than 1.5 dB of error.

Figure 2 shows some representative targets, although not all targets were similarly symmetric about 180°. Due to the size of the data set, the entire process was automated, including target generation. Generation of the data took many weeks on a dedicated Pentium-class processor.

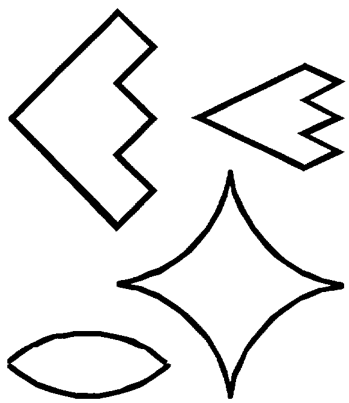

Figure 2. Representative test objects.

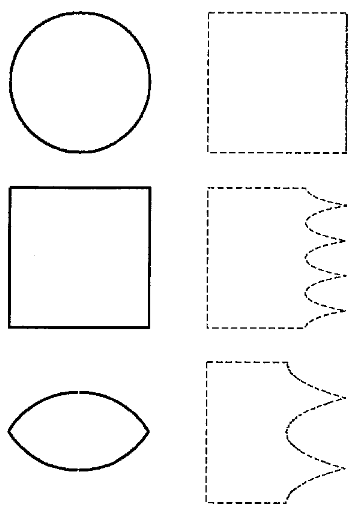

Figure 3. Some simple shapes and their rotational projections.

3. Metric Theory and Calculation

The theory behind the metric will be presented for two-dimensional objects; extension to three dimensions is straightforward. From a heuristic perspective, the *simplest* shape is a circle. It happens to also be a shape for which the MBET is reliable for rather wide bistatic angles. A method is then desired for measuring the deviation from a circular shape that increases monotonically with a subjective definition of complexity. Circles should return the lowest value for any object with a given minimum cross-range dimension. Squares should return nominally higher values, and shapes with interior cavities should return higher values still. Both the measure of complexity and the cross-range dimension are needed for MBET accuracy predictions.

The method used for quantifying complexity is to find the area enclosed by the −3 dB points on the central lobe of the Fourier transform (FFT) of the rotational projection. The rotational projection is a two-dimensional shape transformation of a two-dimensional object. The steps to calculate the metric are as follows:

1. The object is facetized and represented by a matrix of `1`s and `0`s. Matrix rows and columns are spaced 0.1λ apart; a `1` is *inside* and a `0` is *outside* the object. Hollow objects are not supported with this algorithm, although simple cavities are.

2. The *solid-object binary matrix* is cropped and the center point is found.

3. The angular resolution necessary to discriminate between the outer rows and columns of the matrix is found.

4. A second matrix is created with the number of rows determined by the angle steps needed to reach 360°. The number of columns is determined by counting 0.1λ steps in radius out to the furthest point on the object.

5. Starting with an angle of 0 pointing straight upward from the center of the *solid-object matrix*, the first row of the new matrix is filled. As the radius steps outward, the new matrix is filled from left to right by testing the closest corresponding point in the *solid-object matrix*. This is demonstrated in Figure 3, which shows some simple shapes and their rotational projections in dashed lines to the right. A circle becomes a rectangle during the shape transform, because for every angle, the radius is the same.

6. The new *rotational-projection matrix* is cropped and its two-dimensional FFT is found. For a circular *solid object* that has a rectangular *rotational projection*, the FFT returns a sharp spike for the central lobe. Any deviation from a rectangular rotational projection will cause a widening of the center lobe.

7. The −3 dB points are found in both dimensions of the FFT center lobe, and the distances from the center point are multiplied together. This gives an estimate of the area under the −3 dB portion of the lobe.

8. Object complexity is defined by this area estimate. The metric value used in the table is the product of this complexity value and the cross-range dimension at each observation angle. While the metric value is dependent on aspect angle, the complexity value itself is a single value for a given target.

Figure 4 shows the relative value of the complexity value, as a circle is morphed into a square by pushing the corners out and reducing the radius of curvature at the corners to a value of 0.

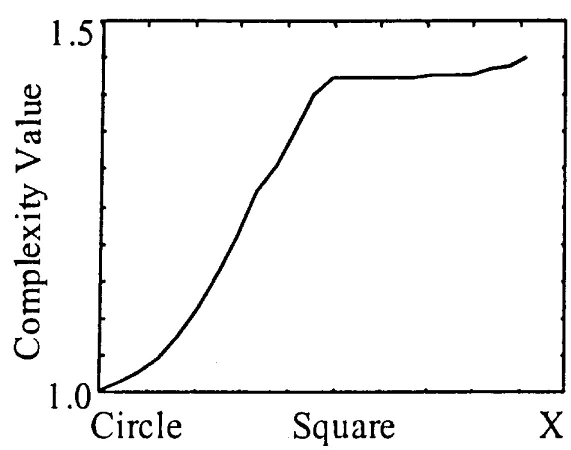

Figure 4. The complexity for various shapes.

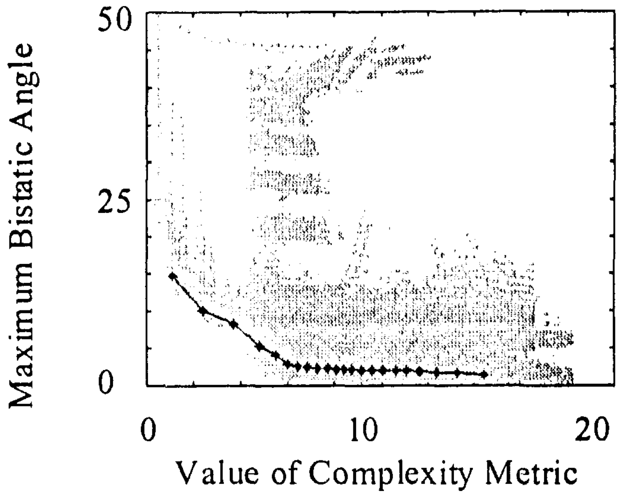

Figure 5. The experimental results; the line shows the 5th percentile.

Table 1. The compiled results.

Metric Value: Cross-Range *x* Complexity	Maximum Reliable Angle Degrees
1.1	14.7
2.4	10.1
3.7	8.2
4.8	5.3
5.5	4.1
6.0	2.9
6.4	2.6
6.8	2.5
7.3	2.4
7.7	2.3
8.1	2.2
8.4	2.2
8.7	2.2
9.1	2.1
9.6	2.1
10.1	2.1
10.6	2.1
11.1	2.1
11.6	1.9
12.4	1.8
13.2	1.8
14.4	1.5

Then, the shape morphing continues to form an X by moving the center part of the square's sides toward the centroid, until the centers of the edges all meet at a single point. The value is generally monotonically increasing, with an intuitive increase in object complexity.

4. Results

The results for the experimental procedure outlined in Section 2 are shown in Figure 5. The gray region is a scatter plot of the maximum angles that were found in Step 4 of the procedure, plotted against the complexity metric from Step 5, and further described in Section 3. The dark line represents the fifth-percentile angle values for metric values, as read along the horizontal axis. This data is repeated in Table 1 for clarity: angle units are degrees for the 95% confidence level of a 1.5 dB worst-case error. The groupings of data points above the main block are due to objects with no concavities, and the points along the top of the plot represent circular or near-circular objects.

5. Extension to Three-Dimensional Objects

To apply this process to three-dimensional objects, only four changes should be necessary:

1. The two-dimensional FFT calculation should be replaced with a three-dimensional FFT.

2. The area calculation of the –3 dB point in the center spike of the two-dimensional FFT should be replaced with a volume calculation in the three-dimensional results.

3. Four maximum angles should be found: increasing and decreasing values of both the azimuth and elevation angles, rather than just the clockwise and counter-clockwise angles found for two-dimensional objects.

4. The two-dimensional projected area will be needed in place of a single cross-range dimension.

Additionally, it is quite likely that some form of wavelet transform could take the place of the FFT, and could provide a more robust basis for the metric.

6. Conclusion

A metric has been proposed that allows reliable prediction of the maximum bistatic angle for which the Monostatic-to-Bistatic Equivalence Theorem (MBET) can be used. It provides a quantitative evaluation of complexity/smoothness and relates this to an angle limit, based on an empirically derived statistical error profile. That is, the metric allows prediction of the maximum bistatic angle for which the MBET provides less than a 1.5 dB error at a confidence level of 95%. Although the metric was only demonstrated for two-dimensional objects at a single polarization and error value, an explanation was provided for easily extending the process to three-dimensional objects, arbitrary polarizations, and alternate error tolerances. This capability allows for optimization of either monostatic collections used for prediction of bistatic data sets, or bistatic computations interpolated to monostatic results.

7. References

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